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Explicit finiteness from the differential equations for the j-function

Specializing a theorem of Orr one obtains the following statement: if $X \subseteq \mathbb{A}^n$ is a subvariety of affine n space over the complex numbers and $\Lambda \subseteq \mathbb{A}^n(\mathbb{C})$ is an isogeny class, then the Zariski closure of $X(\mathbb{C}) \cap \Lambda$ is a finite union weakly special varieties, that is, a component of a variety defined by equations of the form $x_i = \zeta$ and $F_N(x_j, x_k) = 0$ where F_N is a modular polynomial. At the ERC meeting in Cetraro last year I explained how for a fixed isogeny class Λ, even though Orr's proof makes essential use of ineffective ingredients, one can deduce the existence of uniform bounds on the degree of the Zariski closure of $\Lambda \cap X(\mathbb{C})$ as function of the degree of X from general principles.

Barry Mazur has asked to what extent these bounds may be computed explicitly. Using differential algebra, we will compute such explicit upper bounds, independent of Λ, under the hypothesis that for any $x = (x_1, \ldots, x_n) \in \Lambda$ each component x_i is transcendental. The method we employ is quite general and may be extended to other situations in which the set of special points satisfies a differential equation.

(This a report on joint work with James Freitag, presented in the article Strong minimality and the j-function, arXiv:1402.4588.)