David Masser
Universität Basel

The unlikelihood of integrability in elementary terms

For any complex number λ the function $\frac{1}{\sqrt{x^2+\lambda}}$ can be integrated (with respect to x) in "elementary terms" involving logarithms and exponentials. But not $\frac{1}{\sqrt{x^3+\lambda}}$ unless $\lambda = 0$. In 1981 James Davenport claimed that an arbitrary such algebraic $f(x, \lambda)$ can be integrated only for at most finitely many special complex values λ (unless it can be integrated for a general value of λ). Umberto Zannier and I have recently completed the proof of this when f is defined over the algebraic numbers. I sketch the main strategy (through relative Manin-Mumford for abelian varieties) and indicate some interesting features (among other things the splitting properties of certain additive extensions of elliptic curves).