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On two conjectures of Rényi, Erdös and Schinzel about lacunary polynomials

In 1949, Rényi and Erdös independently conjectured that given a polynomial $g(X)$ over the complex numbers, if the square of $g(X)$ is lacunary, i.e., it has a bounded number of terms, then there is a bound on the number of terms of $g(X)$ itself. Schinzel proved it in 1987, actually with any power in place of the square, and he asked whether the same is true for the composition $f(g(X))$, where $f(Y)$ is another given polynomial. This was finally proved by Zannier in 2008.

In a joint work with C. Fuchs and U. Zannier, we extend the result to the following more general case: if $g(X)$ is the root of a polynomial $F(Y)$ of bounded degree, and whose coefficients are themselves polynomials in X with a bounded number of terms, then $g(X)$ is at least the ratio of two polynomials with a bounded number of terms. This can be shown to imply the previous statements. In turn, this general, combinatorial version can be reinterpreted in a geometric fashion as a "multiplicative" version of Bertini's irreducibility theorem, and in several other ways as well.