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Effective uniformity of André-Oort type

The André-Oort Conjecture (AOC) states that the irreducible components of the Zariski closure of a set of special points in a Shimura variety are special subvarieties. Here, a special variety means an irreducible component of the image of a sub-Shimura variety by a Hecke correspondence. The AOC is an analogue of the classical Manin-Mumford conjecture on the distribution of torsion points in abelian varieties. In fact, both conjectures are special instances of the far-reaching Zilber-Pink conjecture(s).

I will present an approach to the AOC that goes back to Yves André himself: before the model-theoretic proofs of the AOC in many cases by the Pila-Wilkie-Zannier approach, André presented in 1998 the first proof of AOC in a non-trivial case, i.e. a product of two modular curves. In my talk, I discuss several results in the style of André's method, allowing to compute all special points in a non-special curve contained in a product of two modular curves.

These results are effective - in stark contrast to those obtainable by the Pila-Wilkie-Zannier approach - and sometimes have also the further advantage of being uniform in the degrees of the curve and its field of definition. An emphasis is put on this aspect of uniformity.