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Answer to some question by Fujita on Variation of Hodge structures (the direct image of ω needs not be semiample)

In the first part of my talk, which shall describe joint work with Michael Dettweiler, I shall provide details for a theorem announced by Fujita 34 years ago, and explain its relation with the Arakelov theorem.

Theorem 1. If one has a Kaehler family fibred over a curve B, then the direct image V of the relative dualizing sheaf ω is the direct sum of an ample vector bundle A and of a unitary flat vector bundle W.

I shall then describe a counterexample to a question raised by Fujita 31 years ago.

Question: Is V semiample?

In view of the previous theorem, the question is whether the flat bundle W corresponds to a finite representation of the fundamental group. While the answer is yes (Deligne) for a summand of W of rank one, or if the base has genus at most one, we show examples, based on hypergeometric integrals, where we get a representation of infinite order, hence

Theorem 2. There are curve fibrations for which V is not semiample.