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CM-points on straight lines

(joint work with B. Allombert and A. Pizarro-Madariaga)

A CM-point (or special point) in \mathbb{C}^2 is a point of the form $(j(\tau_1), j(\tau_2))$, where both τ_1 and τ_2 are imaginary quadratic numbers. A special curve is a plane curve which either a vertical or a horizontal straight line, or the curve defined by $\Phi_m(x_1, x_2) = 0$, where Φ_m is the m -th level modular polynomial. In 1998 Yves André proved that a non-special irreducible plane curve may have only finitely many CM-points. This was the first non-trivial contribution to the celebrated André-Oort conjecture. Relying on recent ideas of Lars Kühne, we obtain a very explicit version of this result for straight lines defined over \mathbb{Q} : with “obvious” exceptions, a CM-point cannot belong to such a line. Kühne (and others) proved this for the line $x_1 + x_2 = 1$.